ANALISI MATEMATICA (MT000001)

1. language

Italian.

2. course contents

Coordinator: Prof. PIERPAOLO NATALINI
Year Course: 1st year
Semester: 1st Semester
CFU/UFC: 12
Modules and lecturers:
- MATHEMATICAL ANALYSIS (MT000001) - 12 CFU - SSD MAT/05
Prof. Pierpaolo Natalini

3. BIBLIOGRAPHY

P. Marcellini e C. Sbordone: Elementi di Analisi Matematica uno, Ed. Liguori;

M. Bramanti, C.D. Pagani e S. Salsa: Analisi Matematica 1, Ed. Zanichelli;

P. Marcellini e C. Sbordone: Esercitazioni di Matematica, 1° Vol., Parte Prima, Ed. Liguori;

P. Marcellini e C. Sbordone: Esercitazioni di Matematica, 1° Vol., Parte Seconda, Ed. Liguori;

M. Bramanti: Esercitazioni di Analisi Matematica 1, Ed. Esculapio;
The texts indicated above are not mandatory; it is up to the student to choose a reference text from among those recommended. The student will be provided with lecture notes, both theory and practical, written by the teacher relating to the topics covered in the course.

4. LEARNING OBJECTIVES

Knowledge and understanding

The student must demonstrate that he has assimilated the fundamental concepts (definitions, statements of theorems, geometric and physical interpretations, any proofs) relating to infinitesimal, differential and integral calculus for the real functions of a real variable and the calculation tools developed. Furthermore, he must be able to understand similar topics by processing the notions acquired.

Applying knowledge and understanding

The student must be able to apply what has been learned in theory through the resolution of verification exercises developed by the teacher, generally linked to topics such as: fields of existence, limits of sequences and functions, numerical series, function studies, definite and indefinite integration, equations in the complex field.

Making judgements

The student must be able to recognize and overcome, by exploiting the mathematical notions acquired, an intuitive aspect and a general approach to the individual problem, to arrive at a formal and rigorous theoretical framework. He must acquire the ability to identify suitable tools and the most appropriate procedures for solving specific problems.

Communication skills

The student must acquire the ability to expound with proper language, supporting or refuting thesis also through examples and counterexamples. Furthermore, you must be able to clearly report the results of the proposed exercises in the form of a problem.

Learning skills

The student must acquire the ability to profitably consult textbooks and handouts made available by the teacher. You must also develop the ability to screen other sources of information.

5. prerequisites

To best address the various topics that will be covered in the course, the student must have the mathematical content indicated in the secondary school programmes. In particular, he must possess the fundamental notions of Analytical Geometry; know the techniques for solving equations, inequalities, systems of equations and algebraic, integer, rational, irrational, exponential and logarithmic, trigonometric and inverse circular inequalities, in absolute value. Must have a complete understanding of the definitions and formulas developed in trigonometry

6. TEACHING METHODS

Knowledge and understanding:

the achievement of knowledge and the ability to understand by the student will take place through the direct involvement of the same during the phase relating to the theoretical introduction of a specific topic.

Applying knowledge and understanding:

the student will be involved and followed during the exercises.

Making judgements:

the student must acquire autonomy of judgment through self-evaluation of exercises carried out.

Communication skills:

students will have to interact by discussing the resolution techniques used in the various exercises and the interpretations of the theorems stated to acquire communication skills.

Learning skills:

The student will be stimulated to identify the appropriate resolution strategies for the various problems covered so that he can continue to study mostly in full autonomy.

7. OTHER INFORMATIONS

Each student will be able to agree, directly or by email, a day and time of office for explanations and clarifications.

8. METHODS FOR VERIFYING LEARNING AND FOR EVALUATION

At the end of the course, the student will be assessed through a written test with a score expressed out of thirty. Furthermore, an intermediate test will be held on the first half of the program.

9. program

Numerical sets – Set of non-negative and natural integers. Set of relative integers. Set of rational numbers. Set of real numbers. Subsets of real numbers. Intervals. Majority and minority of a subset. Limited subset (superiorly and inferiorly). Upper and lower bound of a subset. Absolute maximum and minimum of a subset. Axiomatics of real numbers. Axiom of completeness. Spherical neighborhood of a point. Holey spherical neighborhood. Right and left neighborhood of a point. Interior point of a subset. Accumulation point of a subset. Isolated point of a subset. Induction principle.

Real functions of a real variable – Image and counterimage of a number using the function. Surjective, injective and bijective function. Invertibility of a function and inverse function. Graph of a function and possibly of the respective inverse function. Even and odd function. Monotonicity of functions. Invertibility of strictly monotone functions. Limited functions (top and bottom); increasing, decreasing, absolute maximum and minimum, upper bound and lower bound of a function. Elementary functions: numerical sequence, constant function, powers with natural exponent, n-th roots, polynomials, powers with negative, rational and real integer exponents, absolute value, exponential, logarithm, goniometric and relative inverse functions. Composite functions.

Limit of a function – Introduction to the concept of limit of a function at a point and at infinity. Regular and irregular functions for . Definition of convergent and divergent functions for . Definition of left and right limits. Uniqueness limit theorem. Limit comparison theorem. Remarkable limit . Study of the behavior of elementary functions at the extremes of their domain. Limit of a composite function. Limit of a linear combination, of a product, of a ratio of two functions. Limit of a function in the form . Indeterminate forms. Other notable limitations. Vertical, horizontal and oblique asymptotes.

Continuity – Definition of continuous function in a point and a set. Classification of discontinuities. Continuity left and right. Continuity of elementary functions in their domain. Continuity of a composite function. Continuity of a linear combination, product and ratio of continuous functions. Continuity of functions in form . Sign permanence theorem. Intermediate value theorem. Existence theorem of zeros. Weierstrass theorem.

Differential calculus – Differentiability in a point and in a set of a function. Definition of first derivative at a point and in a set of a function. Right and left differentiability. Subsequent derivatives. Geometric interpretation of the derivative at a point, tangent line to the graph of a function at a point. Differentiability of elementary functions in their domain. Theorem on the differentiability of a composite function. Theorem on the differentiability of a linear combination, product and ratio of functions. Theorem on the differentiability of an inverse function. Differentiability and continuity of a function at a point. Relative extrema of a function (relative max and min). Fermat's theorem. Rolle's theorem. Lagrange's theorem. De l'Hospital's theorem. Criterion for establishing differentiability at a point of continuity of a function. Classification of non-differentiable points: angular, cusp and inflection points with a vertical tangent. Theorem on functions with zero derivative in an interval. Theorem on the strict monotonicity of a function in an interval based on the sign of the first derivative. Finding the relative extrema of a function. Concavity and convexity of a function in an interval. Inflection points. Theorem on the concavity and convexity of a function in an interval based on the sign of the second derivative. Search for inflection points. Study of the graph of a function.

Integral calculus – Definition of definite integral of a continuous function. Its geometric interpretation. Linearity theorem. Additivity theorem. Monotonicity theorem. Modulus theorem. Mean theorem. Weighted average theorem. The fundamental theorem of integral calculus. Definition of primitive function of a given function and related properties. The fundamental theorem of integral calculus. Indefinite integrals. Immediate integrals. Hyperbolic functions and their inverses. Integration by substitution method. Integration method by parts. Integration of rational and irrational functions. Notes on ordinary differential equations.

Numerical series – Definition of numerical series. Convergent, divergent and irregular series. Telescopic series and geometric series. Behavior of monotonic sequences (bounded and not). Cauchy convergence criterion for numerical sequences and series. Necessary condition for convergence of a series. Harmonic series. Linearity properties of two convergent series.

Convergence criteria for numerical series with definitively constant sign terms: comparison criterion, root criterion, ratio criterion, integral criterion, asymptotic comparison criterion. Generalized harmonic series. Series with terms of alternate sign. Leibniz convergence criterion. Absolutely convergent numerical series. Convergence of series from absolute convergence.

Taylor formula – Taylor polynomial of degree *n* of a function with initial point . MacLaurin polynomials of elementary functions. Taylor formula of order *n*. Lagrange representation of the remainder in Taylor's formula. Applications of the Taylor formula in approximation problems. Landau symbol “*o* small” and its operational properties. Representation of the remainder with the Landau symbol *o*. Applications of Taylor's formula in limit calculation problems.

Complex numbers – Definition of complex number. Sum and product of complex numbers. Real and imaginary part. Geometric representation of a complex number. Form, conjugated, main argument. Imaginary constant. Cartesian representation of a complex number. Trigonometric representation of a complex number. Calculation of the main argument as a function of the real and imaginary part. Relationship between complex numbers. Exponential in the complex field. Euler's formula. Exponential representation of a complex number. Power of a complex number. *n*-th root of a complex number. Equations and inequalities in the complex field.